1. Important Inequalities

Exercise 1. Give a distribution for a random variable where the expectation is $1/2$ and the probability that it is greater than or equal to 1 is $2/3$.

Exercise 2. A spider is expecting guests and wants to catch 500 flies for her dinner. Exactly 100 flies pass by her web every hour. Exactly 60 of these flies are quite small and are caught with probability $1/6$ each. Exactly 40 of the flies are big and are caught with probability $3/4$ each. Assume all fly interceptions are mutually independent.

We are trying to find a bound on the probability that the spider catches at least 500 flies in 10 hours.

1. What would the Markov bound be on the probability that the spider will catch her quota of 500 flies?
2. What would the Chebyshev bound be on the probability that the spider will catch her quota of 500 flies?

2. Normal Distribution

Exercise 3. The average life of a certain type of engine is 10 years, with a standard deviation of 3.5 years. The manufacturer replaces free all engines that fail while under guarantee. If he is willing to replace only 2% of the engines, how long a guarantee should he offer? Assume a normal distribution.

3. Central Limit Theorem

Exercise 4. The Cal basketball team plays 100 independent games, each of which they have probability 0.8 of winning. What’s the probability that they win at least 90?