

## CS 70 FALL 2007 — DISCUSSION #3

ASSANE GUEYE, LUQMAN HODGKINSON, AND VAHAB POURNAGHSHBAND

### 1. ADMINISTRIVIA

- (1) Course Information
  - Reminder: The third homework is due September 20<sup>th</sup> at 5pm in 283 Soda Hall
- (2) Discussion Information
  - The second homework is graded and will be distributed in section.

### 2. WELL-ORDERING PRINCIPLE

**Exercise 1.** Use the well-ordering principle to show that if  $x$  and  $y$  are real numbers with  $x < y$ , then there is a rational number  $r$  with  $x < r < y$ .

### 3. THE STABLE MARRIAGE PROBLEM

Recall from class the Stable Marriage Problem, and the associated propose and reject (a.k.a. the Traditional Marriage) algorithm. The following facts can be proven about the correctness of this algorithm:

**Facts 1.** *For the case when men propose and women accept/reject:*

- (i) *No man can be rejected by all women.*
- (ii) *The algorithm terminates with a stable matching.*
- (iii) *The propose-reject algorithm terminates in at most  $n^2$  days.*
- (iv) *The propose-reject algorithm always produces a male-optimal stable matching.*
- (v) *A male-optimal stable matching is a female-pessimal stable matching.*

**Exercise 2.** Try to recall the proof of each of these facts.

**Exercise 3.** Consider an instance of the stable matching problem in which there exists a man  $m$  and a woman  $w$  such that  $m$  is ranked first on the preference list of  $w$  and  $w$  is ranked first on the preference list of  $m$ . Does every stable matching  $S$  for this instance have to contain the pair  $(m, w)$ ?

**Exercise 4.** In a large group of  $n$  men and  $n$  women, Bob, one of the men, gets tipped off that he is the second-highest preference on every woman's list. Bob is pretty happy to hear this. Assuming the traditional (male-optimal) algorithm, might Bob be in for a disappointment? In particular, is it possible that he will end up with the woman he prefers the least of all?

**Exercise 5.** Consider the following situation. After we find the pairing using the male-optimal algorithm we described in class, one of the men  $m$ , who got paired up with some woman  $w$ , becomes more fond of  $w$ , that is, he changes his preference by moving  $w$  higher up in his list. Is the pairing still male-optimal?