1. Applications of Graphs

Exercise 1. Formulate each of the following problems as graph problems; in each case, say whether the graph is directed or undirected, and say what property or structure of the graph one needs to discover.

1. Finding one’s way out of a maze
2. Figuring out an order in which to assemble a complex piece of machinery given that certain sub-parts have to be assembled before others are begun
3. Finding the shortest (in mileage) route from SF to LA
4. Pairing up a bunch of $n$ boys and $n$ girls for a prom dance so that only people who know each other are paired up.
5. Detecting an arbitrage opportunity in the stock market (i.e., a sequence of buy/sell trades that, if all executed, yield a profit)

2. Eulerian paths and circuits

Exercise 2. For which $n$ does the $n$-dimensional hypercube have an Eulerian circuit? How about an Eulerian path?

Exercise 3. For which $n$ does the complete graph $K_n$ have an Eulerian circuit? What about the complete bipartite graph $K_{m,n}$?

3. Trees

Exercise 4. A tree is a connected graph with no cycles. Show that any tree on $n$ vertices has exactly $n - 1$ edges.

Exercise 5. Let $G$ be any connected graph. Design an algorithm to find a spanning tree of $G$. 

Date: October 17, 2007.