CS 70 FALL 2007 — DISCUSSION #8

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1. PROBABILITY

Exercise 1. There are 4 different coins in a box. The probability of Heads when flipping the i-th coin is $1/i$ for $1 \leq i \leq 4$. A coin is selected from the box randomly, and gets tossed until a Head appears.
 (1) Write down a probability space for the experiment.
 (2) Verify that the sum of the probabilities of the sample points is 1.

Exercise 2. Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

Exercise 3. A box contains tickets numbers $1, 2, ..., N$. m tickets are drawn with replacement. What is the probability that the largest number drawn is k?

Not Rule. $\Pr\left(\overline{E}\right) = 1 - \Pr\left(E\right)$

Exercise 4. What is the probability of a head appearing when tossing a fair coin five times.

Inclusion/Exclusion Theorem. $\Pr\left(E \cup F\right) = \Pr\left(E\right) + \Pr\left(F\right) - \Pr\left(E \cap F\right)$

Exercise 5. A card is drawn randomly from a deck of ordinary playing cards. You win \$ 10 if the card is a spade or an ace. What is the probability that you will win the game?

Conditional Probability. $\Pr\left(E \mid F\right) = \frac{\Pr(E \cap F)}{\Pr(F)}$

Exercise 6. A fair coin is tossed twice.
 (1) What is the probability of getting two heads?
 (2) What is the probability of getting two heads given that the first time tail appeared?
 (3) What is the probability of getting two heads given that the first time head appeared?

Independence. $\Pr\left(F \cap E\right) = \Pr\left(F\right)\Pr\left(E\right)$

Exercise 7. Suppose A and B are independent events from a sample space. Prove or disprove:
 (1) $\overline{A}$ and $\overline{B}$ are necessarily independent.
 (2) A and $\overline{B}$ are necessarily independent.