# CS 70 FALL 2007 — DISCUSSION #9

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## 1. Conditional Probability

**Conditional Probability.** $\Pr(E \mid F) = \frac{\Pr(E \cap F)}{\Pr(F)}$

**Independence.** $\Pr(F \cap E) = \Pr(F)\Pr(E)$

**Exercise 1.** Suppose $A$ and $B$ are independent events from a sample space. Prove or disprove:

(1) $\overline{A}$ and $\overline{B}$ are necessarily independent.
(2) $A$ and $\overline{B}$ are necessarily independent.

**Inclusion/Exclusion Theorem.** $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$

**Exercise 2.** A card is drawn randomly from a deck of ordinary playing cards. You win \$ 10 if the card is a spade or an ace or is black. What is the probability that you will win the game?

**Product Rule.** $\Pr(E \cap F) = \Pr(E \mid F) \times \Pr(F)$

**Exercise 3.** Shuffle a deck of 52 cards. What is the probability that the first three cards are aces?

**Bayes Rule.** $\Pr(E \mid F) = \frac{\Pr(F|E)\Pr(E)}{\Pr(F)}$

**Exercise 4.** There are two bags of marbles. The first one contains 30 white and 10 black marbles, and the second one contains 20 marbles of each color. You pick a bag and a marble from it at random and you observe it is red, what is the probability that you picked the first bag of marbles?

**Exercise 5.** In a binary communication channel the receiver sends *zero* or *one*, but at the receiver there are three possibilities: a *zero* is received, a *one* is received, and an *undecided bit* is received (which means that the receiver will ask the transmitter to repeat the bit). Define the event $T_1$={ 1 is sent } and $T_0$={ 0 is sent } and assume that they are equally probable. At the receiver we have three events: $R_1$={ 1 is received }, $R_0$ = {0 is received }, $R_u$={ cannot decide the bit }. We assume that we have the following conditional probabilities: $\Pr(R_0 \mid T_0) = \Pr(R_1 \mid T_1) = 0.9, \Pr(R_u \mid T_0) = \Pr(R_u \mid T_1) = 0.09$.

(1) Find the probability that a transmitted bit is received as *undecided*.
(2) Find the probability that a bit is received in error (error means sending one while receiving zero OR sending zero while receiving one).
(3) Given that we received a *zero*, what is the conditional probability that a *zero* was sent? What is the conditional probability that a *one* sent?

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