1. Administrivia

(1) Course Information
  • HW#4 is graded and ready to hand back today
  • Today’s Midterm Review Session: 5-7pm, 306 Soda.

2. Handshaking Lemma

Let $G = (V, E)$ be an undirected graph. Let $\deg_v$ be the degree of $v$. Then:

**Theorem 1** (Handshaking Lemma).

$$\sum_{v \in V} \deg_v = 2|E|$$

**Exercise 1.** In a group of $n$ people, each person shakes the hand of 3 different people. Prove that $n$ must be even.

**Exercise 2.** The number of vertices of odd degree in a graph $G$ must be even.

3. Induction and Graphs

**Theorem 2.** The number of edges in a tree with $n$ nodes is always $n - 1$.

**Exercise 3.** Prove Theorem 2 by induction.

**Exercise 4.** Use induction to prove that a simple graph with $2k$ vertices and no triangles has at most $k^2$ edges. Give an example where this upper bound is achieved.

4. Eulerian Graphs

**Theorem 3.** An undirected (respectively directed) graph has an Eulerian tour iff every non-isolated vertex has even degree (respectively iff every vertex has equal in- and out-degree).

**Exercise 5.** What is an easy procedure of adding edges to use to make an initial binary tree to have an Eulerian Tour?

**Exercise 6.** Can you always create an Eulerian Tour by adding edges to a tree?

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5. Hypercubes

**Theorem 4.** In $H_n$ (the $n$-hypercube) to isolate any set $S$ of vertices $|S| \leq 2^{n-1}$ you need to cut $|E_S| \geq |S|$ edges.

**Exercise 7.** When can a hypercube also have an Eulerian Tour?