

CS 70 SPRING 2007 — DISCUSSION #7

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1. ADMINISTRIVIA

(1) Course Information

- HW#4 is graded and ready to hand back today
- Today's Midterm Review Session: 5-7pm, 306 Soda.

2. HANDSHAKING LEMMA

Let $G = (V, E)$ be an undirected graph. Let $\deg_v$ be the degree of v . Then:

Theorem 1 (Handshaking Lemma).

$$\sum_{v \in V} \deg_v = 2|E|$$

Exercise 1. In a group of n people, each person shakes the hand of 3 different people. Prove that n must be even.

Exercise 2. The number of vertices of odd degree in a graph G must be even.

3. INDUCTION AND GRAPHS

Theorem 2. *The number of edges in a tree with n nodes is always $n - 1$.*

Exercise 3. Prove Theorem 2 by induction.

Exercise 4. Use induction to prove that a simple graph with $2k$ vertices and no triangles has at most k^2 edges. Give an example where this upper bound is achieved.

4. EULERIAN GRAPHS

Theorem 3. *An undirected (respectively directed) graph has an Eulerian tour iff every non-isolated vertex has even degree (respectively iff every vertex has equal in- and out-degree).*

Exercise 5. What is an easy procedure of adding edges to use to make an initial binary tree to have an Eulerian Tour?

Exercise 6. Can you always create an Eulerian Tour by adding edges to a tree?

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5. HYPERCUBES

Theorem 4. *In H_n (the n -hypercube) to isolate any set S of vertices $|S| \leq 2^{n-1}$ you need to cut $|E_S| \geq |S|$ edges.*

Exercise 7. When can a hypercube also have an Eulerian Tour?