CS 70 SPRING 2007 — DISCUSSION #8

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1. Administrivia

(1) Course Information
- Midterm 1 statistics: Mean 42.8, Standard deviation 7.
- You can pick up your midterm at the end of the discussion section if you haven't done it so.
- Homework 6 is now posted, and it's due on *Wednesday*, March 14, at 2:30pm.

2. Counting

Product Rule. Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are $n_1 n_2$ ways to do the procedure.

Exercise 1. How many different bit strings are there of length eight?

Exercise 2. Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit.

(1) How many possible passwords are there?
(2) What if each password must contain at least one digit?

Exercise 3. How many bit strings of length ten either begins with three 0s or ends with two 0s?

Permutations. $P(n,k) = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$.

Exercise 4. How many permutations of the letters ABCDEFG contain the string BCD?

Combinations. $C(n,k) = \binom{n}{k} = \frac{n!}{(n-k)!k!}$.

Exercise 5. How many bit strings of length eight contain exactly three 1's?

Exercise 6. A club has 25 members.

(1) How many ways are there to choose four members of the club to serve on an executive committee?
(2) How many ways are there to choose a president, vice president, secretary, and treasurer of the club?

Date: March 9, 2007.

3. Combinatoral Proofs

A *combinatoral proof* is a proof that uses counting arguments to prove a theorem, rather than some other method such as algebraic techniques.

Exercise 7. Show that for all integers $n \geq 1$, $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$ holds,

(1) Using induction.
(2) Using a combinatoral proof.

4. Combinations with Repetition

Exercise 8. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$ where x_1, x_2, x_3, and x_4 are nonnegative integers?